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A Broader and More Realistic Game: The Defense of Game Concepts in Evolutionary Game Theory and Individual Based Modeling

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ABSTRACT

Evolutionary game theory (EGT) has been controversial on some conceptual issues due to its own interdisciplinary complexity. The philosopher Liu Chuang believes that the concept of "game theory" should be excluded from the evolutionary game theory model in the field of biology. In addition, because of mathematical equivalence, it is pointed out that the Lotka-Volterra equation is more fundamental than the replicator equation. Zhang Mingjun proposed a thin definition of "game theory" and defended the methods of evolutionary game theory. Liu Chuang's argument lacks rigor both in terms of premise and proof process. Liu Chuang's attempt to reduce evolutionary game theory to ecological equations not only fails to solve the definition problem but also complicates the connotation of "evolutionary games." Since game theory always represents the broad and real interactive scenarios in real life, defining the term "game" is not the primary concern. Instead, the legitimacy of the analogical relationship between the model and the interactions of biological individuals in the real world is more critical.

KEYWORDS

Evolutionary game theory; Replicator equation; Lotka–volterra equation; Game concept; Individual-based modeling; Conceptual engineering

1 Evolution and Game Theory

Game theory is focused on humans' market behaviors and applies to situations in which players must consider others' actions to maximize their own utility (Neumann & Morgenstern, 1944). Like classical game theory, an evolutionary game includes players, strategies, and payoffs, and it is mainly used for studying the evolution of biological phenotypes (Smith, 1982, P.1). They are mainly different in three aspects (Zhang, 2025, p.2):

(1) The player is not a "rational player." In evolutionary game theory, players cannot actively choose their own strategies. They are generally some kind of animals without the ability to choose, or even their genes directly determine their strategies, and genotypes or strategies are "transmitted" through inheritance.

(2) Unlike traditional game theory players who gain "utility," evolutionary game theory participants generally gain changes in "fitness."

(3) There are completely different goals between the two. The goal of classical game theory is to find the "final strategy, " while evolutionary game theory is to analyze the changes in the proportion of individual organisms adopting different strategies in the population that the results of games with different strategies will lead to.

For instance, the Hawk-Dove game (HDG) is a classical scenario mentioned by Smith (Smith, 1982, p.11)[4] and shown in Table 1. Hawks and doves are two kinds of individuals who take different strategies when they discover resources. "V" is the total resources. When hawks fight versus hawks, they will fight for resources, and they both have a 1/2 possibility to win all of the resources. "C" represents the price they will pay for the fight. If it's Doves versus Hawks, Doves will always choose escaping, so Hawk will get all V. If a Dove encounters a Dove, they will share all the resources equally.

Table 1 The payoff matrix of the "eagle-pigeon game" (Smith, 1982, p.12)

	Hawk	Dove
Hawk	(V-C)/2, (V-C)/2	V,0
Dove	0,V	V/2, V/2

Using the traditional method to solve this question, the Nash equilibrium is both players choosing Dove. However, assume there are only doves in this population, but a hawk appears due to genetic mutations. Then, Hawk can only versus with a Dove, but it is obvious that the fitness of the Hawk is always greater than the Dove, but when the Dove population increases, the payoff of Hawk versus Hawk is smaller than Dove versus Dove, so because the population ratio of species has been changing over time, the situation of the game is also constantly changing. It is very difficult for us to study a game in a biological context using a static analysis (the definition of Evolutionary Stable Strategy and the method for analyzing the eagle-pigeon game are supplemented in Appendix A). Therefore, it is very important to study the trend of population proportions of different strategies changing over time using dynamic equations. The most classic and commonly used dynamic equation is the replicator equation：

$$\dot{x}_i = x_i[f_i(\mathrm{x}) - \bar{f}(\mathrm{x})] \tag{1}$$

In the second part, the mathematical relationship between the replicator equation and the LV equation will be introduced, as well as Liu Chuang's rebuttal to the connotation of "game" in the biological context. The third part will analyze the mistakes made by Liu Chuang in his argument. The fourth part will analyze why the LV equation cannot replace the replicator equation and propose that evolutionary game theory is actually based on the analysis results of individuals.

2 Mathematical Equivalence and the Connotation of Game

To describe a certain moment "t" as a whole, *x*i represents the frequency of species occurrence (or the proportion in the population), and the vector state x(t) = [x1(t), x2(t),..., xn(t)] is used for the overall description. The return is A = [aij], and the expected return of strategy i is (Ax)i, where i = 1, 2,... "n. The average return is frequency multiplied by expectation: $\mathrm{x}\cdot A\mathbf{x} = \sum_{i,j}^{n} a_{ij}x_ix_j$. (Liu, 2021, p.4).

If return is substituted into the replicator equation (1), this formula can be obtained:

$$\dot{x}_i = x_i[(Ax)_i - (\mathrm{x}\cdot A\mathrm{x})] \tag{2}$$

Hofbauer proved that the replicator equation could be transformed into the format of the LV equation through mathematical transformation. LV equations describing the dynamics of interacting populations of predators and prey created by Vito Volterra (Hofbauer & Sigmund, 1998, p.11):

$$\frac{\mathrm{d}V}{\mathrm{d}t} = rV - (aV)P$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = b(aV)P - mP \tag{3}$$

Among them, V is the population density of the prey. P is the population density of the predator, r is the growth rate of the prey, m is the mortality rate of the predator, and a and b are the response coefficients (Zhang, 2025, p.6).

When simulating the interactions of n species, the general form of LV equations is:

$$\dot{x}_i = x_i\left(r_i + \sum_{j=1}^{n} a_{ij}x_j\right), \quad i = 1, \dots, n \tag{4}$$

Indeed, equations (1)-(4) is equivalent because "There exists a differentiable and invertible mapping from Sn = {x = (x1, ..., xn)∈Rn} to $\dot{x}_i = x_i[(Ax)_i - x\cdot Ax)], \quad i = 1, \dots, n$, which transforms the trajectory of the replicator equation ẋ = xi[(Ax)i - x·Ax)] into the trajectory of the LV equation

$$\dot{y}_i = y_i\left(r_i + \sum_{j=1}^{n-1} b_{ij}y_j\right), \quad i = 1, \dots, n-1 \tag{5}$$

and $r_i = a_{in} - a_{nn}$, $b_{ij} = a_{ij} - a_{nj}$ (Liu, 2021, p.6)."

After summarizing the arguments of J. Hobauer and K.Sigmund, Liu Chuang further demonstrated why mathematical structures should be used to replace the "game" meaning for the application of evolutionary game theory in the field of biology. Liu Chuang's argument can be simplified as(Liu, 2021, p.7-p.9):

P1: In the context of biology, evolutionary game theory does not have the true meaning of games.

SP1.1 In the field of biology, players in the evolutionary game theory cannot "make choices".

SP1.1.1 Animals or some non-living things cannot make choices after "careful consideration".

SP1.2 The benefit of organisms with a certain phenotype in the game is the improvement of fitness rather than utility in the general sense.

P2: The replicator equation in evolutionary game theory can be written in a "simpler and more fundamental" Lotka-Volterra equation format .

C1: The application of evolutionary game theory in the field of biology should completely abandon its game theory connotation.

3 The Conceptual Problems Committed by Liu

In fact, neither the premise nor the Inference of Liu's argument mentioned in part 2 holds true and valid.

Liu's arguement mainly focus on SP1.1. He distinguishes the "players" under evolutionary context and social science context because the animal in the evolutionary context usually cannot make a choice after "careful consideration (Liu, 2021, p.9)", and he mainly use Rpeated Prisoner's Dilemma (RPD) as an example.

However, this proposition is more complex than Liu originnally thought. First of all, "careful consideration" is unclear and metaphorical. The differences between prisoner and the hawks and doves in HDG into two points is:

(1) Prisoners in the RPD have "intentionality".

(2) The mechanism by which prisoners change their strategies is "learning", while hawks and doves do so through

natural selection.

Firstly, in philosophy, intentionality is defined as the ability of consciousness to point to the external world in some way (Husserl, 2012, p. 19). However, in recent years, as physicalism has gradually gained popularity, this definition has be reproached inapplicable. Although this definition seems very simple, there is no objective standard to examine wether an object have intentionality. Thefore, more philosopher abandones the ontological explannation on intentionality. For instance, Daniel Dennett believes that intentionality is merely a explanation and position (Denett, 1989, p. 13-43). For example, if we say that people have intentionality, it will be easier to provide theoretical explanations for human behavior in daily life, but it has no substantive connotation. Intentionality is difficult to translate into some kind of physical or objective description. Sims proposed a alternative solution to define intentionality, that is, to replace the criterion of intentionality with whether a certain object has conceptual ability, and to transform "whether it has conceptual ability" into the question of "whether it can make judgments about the future". But the result of this is that Sims believes escherichia colis have intentionality and everything is intentional to some extent, it's just a matter of more or less(Sims, 2021, p.8-p.15). Obviously, this was different from what Liu had in mind. So, I think it won't hold true until Liu gives a strict definition of intentionality.

Secondly, What is the essential difference between learning and natural selection? Natural selection is generally written as three premises (Lewontin, 1970, p.1):

(1) There are different phenotypes

(2) Different phenotypes have different adaptability

(3) Phenotypes can be inherited

In his rebuttal to Liu, Zhang Mingjun has already mentioned that prisoners are not free subjects but rather subjects who act mechanically according to certain rules (Zhang, 2025, p.4). It is significant to distinguish between the theoretical subject and the practical subject. The former is the object fitted in a certain theoretical model, while the latter is the object in the real world. Liu Chuang intuitively believes that learning is different from natural selection because he directly makes judgments based on his daily experience. However, he neglects that “game theory” itself is a theoretical concept, which is based on many of his own theoretical assumptions. And these theoretical premises are no different from the three premises of natural selection. For example, learning behavior can be written as the following three premises:

(1) There are different "behaviors".

(2) Different behaviors have different degrees of adaptability

(3) "Behavior" can be learned

SP1.2 is make a more obvious mistake: there is no reason to assume that the improvement of utility and fitness is contradictory. For instance, the utility that prisoners gain is essentially the time they spend living outside prison, and we have every reason to assume that prisoners prefer to spend more time outside prison because they can thereby enhance their adaptability (with a higher probability of spreading their phenotypes by generating offspring). Similarly, conversely, the improvement of adaptability also brings many side effects, such as more opportunities to taste delicious food and more time spent outside prison. Although they cannot be completely equated logically with the two, at least in many specific cases, utility and fitness can be used interchangeably under the proncipe of substitutivity of identicals.

4 Game Theory Based on Individual-Based Modeling

There are unbridgeable gap between replicator equation can be rewritten as the Lotka-Volterra equation and replicator equation can be reducible to Lotka-Volterra equation. To discuss that one of the two can be reduced to the other not only requires formal equivalence, but also demands that one of them can contain the other in connotation. For example, the prerequisite for discussing that the replicator equation can be reduced to the LV equation is that each term of the replicator equation transformed into the form of the LV equation must have precise biological implications. In addition, In order to prove that the LV equation is more fundamental than the replicator equation rather than that the replicator equation is more fundamental than the LV equation, it is necessary to logically explain the "interspecific competition" described by the LV equation is more fundamental than the "game" phenomenon in evolutionary game theory.

Before drawing a conclusion, it is necessary to verify what the LV model is like in specific game scenarios in actual situations. Consider the following situation under the premise assumptions of evolutionary game theory：

Table 2 The payoff matrix between strategy A and strategy B (Zhang, 2025, p.6)

	Strategy A	Strategy B
Strategy A	*a*	b
Strategy B	c	d

f (x,y) represents the fitness of a aginst b, and the formula of u is:

$$f(\mathrm{A,A}) = \mathrm{a} \tag{6}$$

xi represents the proportion of strategic players, and it is also the probability of this game occurring.

The fitness of the species adopting Strategy A and Strategy B is：

$fA = \mathrm{a}xA + \mathrm{b}xB$

$$fB = \mathrm{c}xA + \mathrm{d}xB. \tag{7}$$

The average futness of the population is:

$$\bar{f} = xAfA + xBfB. \tag{8}$$

The replicator equation is：

$$\dot{x}\mathrm{A} = xA\,(fA\,-) \tag{9}$$

$$\dot{x}\mathrm{B} = xB\,(fB\,-) \tag{10}$$

Change the replication equation to the LV equation：

$$y_1^{\cdot} = y_1(r_1 + b_{11}y_1) \tag{11}$$

$r_1=a_{12}-a_{22}$, $b_{11}=a_{11}-a_{21}$

According to Liu Chuang's perspective, after transforming the replicator equation into the LV equation, the equation is：

Some problems have emerged in the process：

(1) Just as Zhang Mingjun pointed out, the replicator equation transformed into the LV equation has undergone a transformation in connotation and is difficult to explain biologically: Originally, the replicator equation was used to analyze the rate of change in the population proportion of species using two strategies. However, after being transformed into the LV equation, only the change in population density of one species over time can be analyzed. Because we can understand AAA as the population proportion of strategy A, then BBB is the population proportion of strategy B. That is to say, the population ratio of a certain strategy is always fixed at 1, and also "$r1=a12-a22$" do not has clear biological meanning (Zhang, 2025, p.7).

(2) If the shapes of two words are similar but their usages are completely different, it is very difficult for us to say that the two words are synonyms. For instance, "beautiful" and "beauty" are adjectives and nouns respectively. Although they look very similar, they should not be confused; otherwise, the choice of words will be very confusing. The LV equation and the replicator equation are completely different in application. For example, in the replicator equation, "a" and "b" represent the gains of the game between two biological individuals. In the LV equation, r_i expresses the coefficients regarding the growth rate and mortality rate of a certain population. So for a modeler, using the replicator equation and using the LV equation mean completely different measurement processes.

At the end of his article, Liu Chuang even said that the next step could be to convert the replicator equation that has already been transformed into the LV equation into the Price equation for dealing with genotype evolution in population genetics (Liu, 2021, p.10). Liu did not respond to the questions listed above regarding reducing the replicator equation to the LV equation. He didn't even focus his discussion on the "reduction" relationship between the two equations. Instead, he pointed out that the two equations could be reduced, and then attacked that evolutionary game theory in the field of biology had no meaning of "game", but the reduction relationship between these two equations remained very dim. Even why the replicator equation is reduced to the LV equation rather than the LV equation being reduced to the replicator equation remains unknown. This view is extremely dangerous because, in fact, in practice, these two equations are very different both in terms of premise assumptions and the specific modeling process. Liu Chuang pointed out the possible problems of the concept of "game" and reduced the replicator equation to an equation that is very different in practice and meaning. This will not only fail to solve the conceptual problems of evolutionary game theory, but also make the conceptual meanings more confusing. So Liu's biggest problem does not lie in an argument or a proposition, but in his belief that an ecological or genetic approach is needed to restore the idea of evolutionary game theory.

Zhang Mingjun has expressed that he believes that the specific individual-based model is the essence of the replicator equation (Zhang, 2025, p.7-8). In fact, whether it is the replicator equation or the LV equation, their basis is the interaction relationship among species in the real world. This is also why Liu Chuang made a mistake in the previous alternative argument, because he failed to understand that the concept of "game" itself is also a theoretical model of the player in the actual situation. In reality, there is no absolute rational subject, and no one will completely imitate the "prisoner" in the Prisoner's dilemma to learn in the game. Wiener and Bateson also expressed the same criticism of von Neumann's game model - human desires are changeable and complex, but they are always singular in game theory. Similarly, the replicator equation is also based on many assumptions, such as that the population is infinitely large... So the most important thing is not to study what other equations the replicator equation can be transformed into, but to study how game theory models under different circumstances generate analogical and simulated relationships with specific situations in reality. It is based on individual modeling that collapses in a certain situation to replicate sub-equations or LV equations.

Appendix 1 Evolutionaryly Stable Strategy

Because of the huge differences between the two, Nash equilibrium is also not applicable to evolutionary game theory. when under the situation below:

Assuming that 1) plarers are all vegetative propagation, 2)the game is happened randomlly, and 3) the player in both sides cannot distinguish themselve from others (The game occurs symmetrically).

Table 3 The payoff matrix between strategy P and strategy Q

Strategy	P	Q
P	a	b
Q	c	d

f (x,y) represents the fitness of a aginst b, and the formula of u is:

x_i represents the proportion of strategic players, and it is also the probability of this game occurring.

$f\,(P,P) = a$

$fA = ax_P + bxQ$

$fB = cxP + dxQ$

The most famous replacement solution of Nash equilibrium is ESS proposed by Smith and Price, and the defination of ESS is if all the individual in a population use a certain strategy, these population will not be invade when some mutants occur. The foermulate written in P is an ESS is and only if (Huttegger, 2013, p.640-641):

(1) $f\,(P, P) \geqslant f\,(Q, P)$ for all alternative strategies s and

(2) if $f\,(P, P) = f\,(Q, P)$, then $f\,(P,Q) > f\,(Q,Q)$.

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